A MATHEMATICAL STUDY OF ANTERIOR DENTAL RELATIONS: PART II, INCISOR AND CANINE OVERJET

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A mathematical model of anterior inter-arch relations was described in a previous article.[1] This model is modified and manipulated and, using hypothetical dental measurements, results are generated and presented. The present work continues the two-dimensional examination of anterior dental relations in the first article, the emphasis remaining on incisor and canine overjet.

Other authors have started by observing ideal occlusions, then have worked backwards to gain ratios of preferred maxillary to mandibular teeth widths; this restricts their studies to Class I occlusions with complete anterior dentitions and similar anterior form of both arches. This paper has no premise of normal or initial ratios. Incorporation of multiple factors allows forming and testing of hypotheses of anterior dental relations.

Many factors influence anterior dental relations in varying degrees. Some dental measurement changes examined here are: the sum of teeth widths in each arch; spacing; crowding; angle of the arc of each arch and the antero-posterior buccal relation.

Summary tables are presented to aid the prediction of the direction of inter-arch response to change in a dental measurement. Some inferences are discussed and are presented as a series of principles considered valid for this model. The principles may warrant testing with other arch form models. (Aust Orthod J 1996;14:143-149)

Key Words : Mathematics, arch form, overjet, tooth width, arc angle, occlusion, dental spacing, dental crowding, dental arch width, dental arch depth.

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Aim

To extend the mathematical model outlined in Part I,[1] in order to examine how altering one or more dental measurements changes other measurements. Some concepts of anterior dental relations may be proposed and tested by examining results of calculations.

Introduction

Neff[2] Bolton[3] and Ho and Freer[4] found that differences in widths of the teeth between the arches creates the likelihood of changes in other dental measurements. Dukes[5] (referring to Pollock's work and teaching), and Epker and Fish[6] describe changes to inter-arch relations by varying both arch forms simultaneously.

In Part I[1] Bonwill's[7,8,9] model was used in the example spreadsheet and remains a reference model. The Bonwill model locates the canine on an arch which allows relations to be established between two arches in the horizontal plane. This article will use graduated ranges of hypothetical measurements to create tables for determining change in incisor and canine overjet when other dental measurements are varied.

Method

This paper maintains the assumptions and formulæ presented in Part I[1] which are contained within a spreadsheet using the multi-dimensional spreadsheet program, Lotus Improv 2.0*. Computer hardware continues as an IBM-compatible with an 80486 CPU. The layout of the spreadsheet in its most reduced form** is given in Table I.

When two arches are regarded together, the observation position can alter the perspective of inter-arch relations. A parallax-type effect may result and is directly analogous to the cone shifting technique for localising unerupted teeth. If the observation point is forward of the reference plane, an adjustment is required when an arch with the greater canine width is related to its narrower opponent. The observation position was taken as perpendicular to the curve of the outer arch in the canine region and is derived from data in Table I, lines 16 and 19, and calculated with Table II, formulæ 14 and 17.

The inter-arch change from the observer position alters the relation of the canines of the opposing arches, with the whole arch being projected in the same direction and by the same antero-posterior amount as the

MAXILLARY PERIMETER	SUM MAX INC + 1C mm	36.00
	MAX SPACING mm	
	TOTAL mm	36.00
MAX ARC IN DEGREES		120
MAX ARC IN RADIANS		2.09
MAX RADIUS		17.19
DEPTH OF MAXILLARY SEGMENT mm		8.59
WIDTH OF MAX SEGMENT mm		29.77
MANDIB PERIMETER	SUM MAND INC + 2C mm	34.00
	MAND SPACING mm	
	TOTAL mm	34.00
MAND ARC IN DEGREES		120.00
MAND ARC IN RADIANS		2.09
MAND RADIUS mm		16.23
DEPTH OF MAND SEGMENT mm		8.12
WIDTH OF MAND SEGMENT mm		28.12
MAX TO MAND	MAX-MAND RAD mm	0.95
	A-P CANINE ADJUSTMENT mm	0.48
	MAX-MAND DEPTH mm	0.48
	INCISOR OVERJET mm	0.95
	LATERAL ADJUSTMENT AT CANINE mm	0.28
	MAX-MAND WIDTH/2mm	0.83
	LATERAL (@CANINE) OVERJET mm	1.10

Table I. The spreadsheet used for all calculations.

* Lotus Improv 2.0 uses a different spreadsheet format or layout. In calculating, there are no set named cells, rather items that are related to each other through categories and the categories can be moved to X, Y or Z axes. Calculations are between items rather than cell addresses but the mathematical principles and formulæ used are the same for both. A multidimensional spreadsheet is a tool for reorganizing information in a spreadsheet. A conventional spreadsheet such as Lotus 123 will calculate the same result with the same formula.

** Interrelated columns and rows are used as in Table 3 but each calculation in these expanded forms is identicfal to multiple calculations of Table 1. For example; Table 3 has eight columns and the same results can be obtained by calculating Table 1 for each of the eight maxillary teeth widths of Table 3.

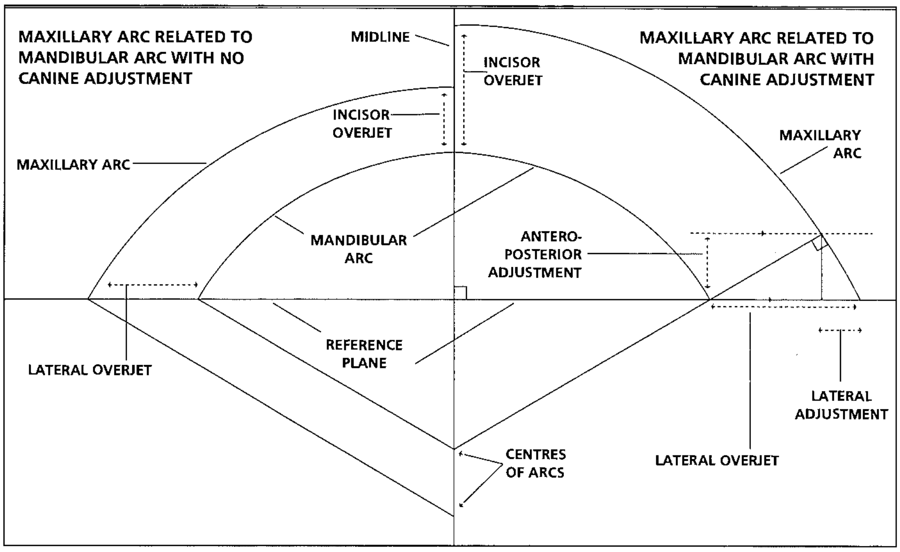


Figure 1. Relating maxillary and mandibular arcs to each other with and without canine adjustment.

1.	MAXILLARY PERIMETER. Total = groupsum (MAXILLARY PERIMETER)
2.	'MANDIB. PERIMETER'. Total = groupsum ('MANDIB. PERIMETER')
3.	:MAX ARC IN RADIANS = :MAX ARC IN DEGREES * @pi / 180
4.	:MAND ARC IN RADIANS = :MAND ARC IN DEGREES * @pi / 180
5.	MAX RADIUS = MAXILLARY PERIMETER. Total / :MAX ARC IN RADIANS
6.	MAND RADIUS = 'MANDIB. PERIMETER'. TOTAL /: MAND ARC IN RADIANS
7.	DEPTH OF MAXILLARY SEGMENT = MAX RADIUS * (1-cos (:MAX ARC IN RADIANS /2))
8.	DEPTH OF MAND SEGMENT = MAND RADIUS * (1-cos (:MAND ARC IN RADIANS /2))
9.	'MAX-MAND RAD' = MAX RADIUS - MAND RADIUS
10.	Incisor Overjet = DEPTH OF MAXILLARY SEGMENT - DEPTH OF MAND SEGMENT + 'A-P Canine Adjustment'
11.	WIDTH OF MAX SEGMENT = MAX RADIUS * 2* sin (:MAX ARC IN RADIANS /2)
12.	WIDTH OF MAND SEGMENT = MAND RADIUS * 2* sin (:MAND ARC IN RADIANS /2)
13.	'Lateral (@canine) overjet' = 'Max-mand width/2' + Lateral adjustment at canine
14.	'A-P Canine adjustment' = IF ('Max-mand width/2' >0, 'Max-mand width/2' *tan (90-:MAX ARC IN DEGREES/2)@pi/180),
	'Max-mand width/2' * tan ((90-:MAND ARC IN DEGREES/2)*@pi/180))
15.	'Max-Mand depth' = DEPTH OF MAXILLARY SEGMENT - DEPTH OF MAND SEGMENT
16.	'Max-mand width/2' = (WIDTH OF MAX SEGMENT - WIDTH OF MAND SEGMENT)/2
17.	Lateral adjustment at canine = IF ('Lateral @canine) overjet' >0, 'A-P Canine adjustment'/tan (:MAX ARC IN RADIANS/2),
	'A-P Canine adjustment'/tan (:MAND ARC IN RADIANS/2))

Table II. Formulæ used in Table I spreadsheets.

canine movement - described in this series as a canine adjustment. If one arch is regarded as static, the other arch is adjusted relative to the static arch. The static arch is fixed with respect to the reference plane. In this paper, the mandibular arch is regarded as static and the maxillary arch is adjusted. There are differences between results of canine adjusted figures and those without adjustment. Canine adjustment may not be appropriate in all circumstances.

Unless otherwise stated, the dental measurements for this paper are:

maxillary perimeter	36mm;
mandibular perimeter	34mm;
crowding or spacing	0(zero)
angle of arc	120deg;
Class II or III trend	0(zero)

A-P adjustment at the canine calculated per Table 2, formula 14.

Lateral adjustment at the canine calculated per Table II, formula 17.

A list of the formulæ* is presented in Table II**. Figure 1 depicts maxillary and mandibular arcs with and without canine adjustment calculations.

Use of a matrix to express the result of varying two measurements

Rather than altering one measurement at a time and observing the response in all other measurements, data of one calculated measurement may be examined against simultaneous variation of multiple dental measurements. Figures 2 to 8 are presented in a matrix form with the features observed being incisor and canine overjet. Figure 2 presents eight different maxillary perimeter lengths (as defined in Part I[1]) and eight different mandibular perimeter lengths, producing sixty-four results for incisor overjet.

* Lotus Improv 2.0 does not use cells but builds relations between items allowing for formulæ to be expressed as relations to named items as listed in Table II.

** The formulæ used with Lotus Improv 2.0 software may need altering for use in other software. The function names may be different and the software may require specific syntax. The 'IF' function in Improv is similate to 'IF, THEN, ELSE' or equivalent function in some other spreadsheets or databases.

Results

Varying the sum of maxillary and mandibular teeth widths

Table III displays variations in maxillary teeth widths in 1mm incremental increases. The changes in maxillary perimeter length are reflected in changes to figures within the maxillary arch and between the two arches. As the maxillary teeth widths increase, the incisor and canine overjets increase. Table III calculations form the top row of data in Figures 2 and 3. For Figures 2, 3, 6, 7 and 8, maxillary measurements are on the X-axis and mandibular measurements on the Y-axis.

Another separate table can be calculated with the maxillary perimeter constant and the mandibular arch perimeter varied. Rather than varying one factor at a time, a matrix was used to vary two factors within the one table. By using the calculation for one measurement, changes become evident and can be analysed visually in the graph and by reference to the table attached to the graph (Figure 2). Trends are easier to follow using the matrices such as those in Figure 2 and all figures following.

Altering both maxillary and mandibular crowding and spacing

Figure 4 graphs incisor overjets with incremental variation in maxillary and mandibular crowding. When both arcs have the same angles, incisor overjet responds in the same direction for increase or decrease in crowding or spacing. The same response occurred for canine overjet.

When crowding or spacing of both arches is equivalent, the overjet relation is the same as for no crowding or spacing; the overjet for 8mm, 4mm and 0 crowding in both arches is the same (incisor overjet=0.95mm in Figure 4) for all. Also, a discrepancy between the arches results in overjets whose magnitude is maintained if the magnitude of the discrepancy is maintained. For example: as shown in the table attached to Figure 4, 8mm maxillary spacing and 4mm mandibular spacing have the same overjets as 4mm maxillary crowding and 8mm mandibular crowding (incisor overjet=2.86mm).

Arch perimeter length

The formula for arch perimeter length involves adding the teeth widths to arch spacing (crowding is subtracted). The resultant perimeter length is the same for an increase in teeth widths of four millimetres as it is for an equivalent space in the same arch.

MAXILLARY TEETH WIDTHS		36	37	38	39	40	41	42	43
MAXILLARY PERIMETER	SUM MAX INC + 1C mm	36.00	37.00	38.00	39.00	40.00	41.00	42.00	43.00
	MAX SPACING mm								
	TOTAL mm	36.00	37.00	38.00	39.00	40.00	41.00	42.00	43.00
MAX ARC IN DEGREES		120	120	120	120	120	120	120	120
MAX ARC IN RADIANS		2.09	2.09	2.09	2.09	2.09	2.09	2.09	2.09
MAX RADIUS		17.19	17.67	18.14	18.62	19.10	19.58	20.05	20.53
DEPTH OF MAXILLARY SEGMENT mm		8.59	8.83	9.07	9.31	9.55	9.79	10.03	10.27
WIDTH OF MAX SEGMENT mm		29.77	30.60	31.43	32.25	33.08	33.91	34.73	35.56
MANDIB PERIMETER	SUM MAND INC + 2C mm	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00
	MAND SPACING mm								
	TOTAL mm	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00
MAND ARC IN DEGREES		120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
MAND ARC IN RADIANS		2.09	2.09	2.09	2.09	2.09	2.09	2.09	2.09
MAND RADIUS mm		16.23	16.23	16.23	16.23	16.23	16.23	16.23	16.23
DEPTH OF MAND SEGMENT mm		8.12	8.12	8.12	8.12	8.12	8.12	8.12	8.12
WIDTH OF MAND SEGMENT mm		28.12	28.12	28.12	28.12	28.12	28.12	28.12	28.12
MAX TO MAND	MAX-MAND RAD mm	0.95	1.43	1.91	2.39	2.86	3.34	3.82	4.30
	A-P CANINE ADJUSTMENT mm	0.48	0.72	0.95	1.19	1.43	1.67	1.91	2.15
	MAX-MAND DEPTH mm	0.48	0.72	0.95	1.19	1.43	1.67	1.91	2.15
	INCISOR OVERJET mm	0.95	1.43	1.91	2.39	2.86	3.34	3.82	4.30
	LATERAL ADJUSTMENT AT CANINE mm	0.28	0.41	0.55	0.69	0.83	0.96	1.10	1.24
	MAX-MAND WIDTH/2mm	0.83	1.24	1.65	2.07	2.48	2.89	3.31	3.72
	LATERAL (@CANINE) OVERJET mm	1.10	1.65	2.21	2.76	3.31	3.86	4.41	4.96

Table III. Expansion of spreadsheet with the sum of maxillary teeth widths increased in 1 mm increments.

Antero-posterior and lateral adjustment in the canine region

The relation of the difference in depth of the arc between the arches and canine adjustment is constant when the angle of the arc is constant. The constancy of the relation between arc depth difference and canine adjustment is explicable because, for a set angle, the trigonometric ratio is fixed. Variation in the angle of the arc alters the trigonometric ratio thereby changing the canine adjustment ratio.

For the Bonwill[7,8,9] model (120 degree arc angle), the canine adjustment for antero-posterior overjet is equal to the difference between maxillary and mandibular arc depths (Table III). For almost all other arc angle values (one obvious exception being when lateral overjet is zero), the difference in depth is different from the canine adjustment, as shown in Figures 6 and 7.

Variation of the buccal relation

Variation of the buccal relation is portrayed in Figure 5. The matrix for Figure 5 describes incisor and canine overjets as the buccal relation is altered from -10 mm to +10 mm in 2mm increments.

Incisor overjet results reflect the formula that adds the incisor overjet at zero to the buccal trend. Lateral overjet increases and decreases at a faster rate than incisal overjet – a large range being reflected in the figures. Lateral overjet may also be altered; some further factors will be addressed in the discussion section.

Varying maxillary and mandibular arcs

Figures 7 and 8 have the maxillary perimeter fixed at 36 mm and the mandibular perimeter fixed at 34 mm, with incremental variation occurring in the angles of maxillary and mandibular arcs. Figure 7 shows incisor overjet with the arcs varied, and Figure 18 shows canine overjet with the arcs varied.

Discussion

Horizontal control of tooth position is an aspect of Dentistry and Orthodontics that should be understood for both ideal and non-ideal occlusions. In addition, it should be possible to predict the consequence of change within one arch in its relation to the other. The mathematical model developed in the present work was tested with hypothetical scenarios and the results allowed some principles to be supported and others proposed.

Patients may display features ranging from narrow to broad arches, and crossbites and antero-posterior relations that they may want corrected. Summary tables are presented later in this section which may allow some forecast of occlusal response to treatment. These tables and the mathematical model have implications for better comprehending initial malocclusions and the ramifications, both favourable and unfavourable, of various treatments. Also, principles are stated that are considered valid for this model.

Arch perimeter length

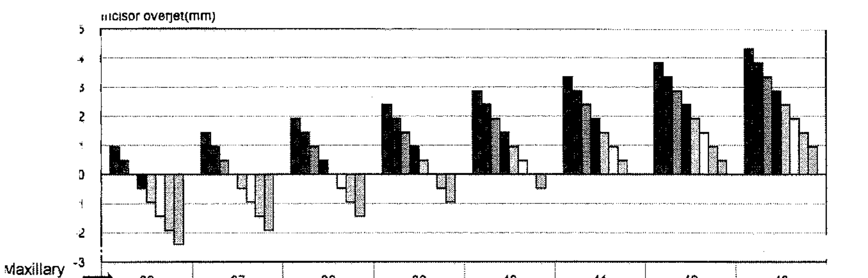
Results for changes in teeth widths, crowding and spacing can be considered together as components of the arch perimeter. Changing any of the three measurements elicits results of the same magnitude because they are subject to the same formulæ. The work of Neff,[2] Bolton[3] and Ho and Freer[4] requires a premise of a complete occlusion with an Angle Class I buccal relation. Dental variations, such as gross tooth size discrepancies, cannot be readily resolved in the aforementioned papers. Missing and supernumerary teeth add complexity in comprehending the features of many malocclusions, and inter-arch ratios do not show all possible solutions.

Crowding presents some difficulties, as it may be expressed as rotation or labio-lingual tooth displacement. Both rotation and labio-lingual displacement create variations in overjet measurements. Crowded teeth may be in either an anterior or posterior position to the line of the arch and this may cause local deviation from a predicted overjet.

Figure 2 illustrates an increase in incisor overjet as maxillary teeth widths increase or mandibular teeth widths decrease. The Table attached to Figure 2 shows a linear pattern of change in the figures. An increase or decrease in both arches by the same amount maintains the same overjet. An increase in the mandibular teeth widths alone results in a decrease in overjet; an increase in maxillary teeth width increases overjet (Tables I and II).

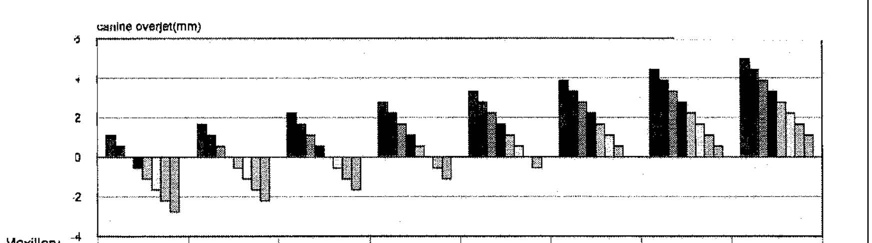
Figure 4 is similar to Figures 2 and 3 in that tooth widths, crowding and spacing effectively alter the arch perimeter. The way in which Figure 4 may be considered more suitable could be as a malocclusion where mandibular crowding and maxillary spacing can result: for example, with a large overjet when other arch measurements are normal. It is also possible for spacing or crowding to counteract a tooth width discrepancy; an example being an anterior occlusion with an acceptable overbite and overjet, yet having small maxillary lateral incisors and some mandibular crowding.

Principle 1: Arch perimeter is the sum of tooth widths with spacing within the arch added; crowding is subtracted from the sum of the tooth widths.



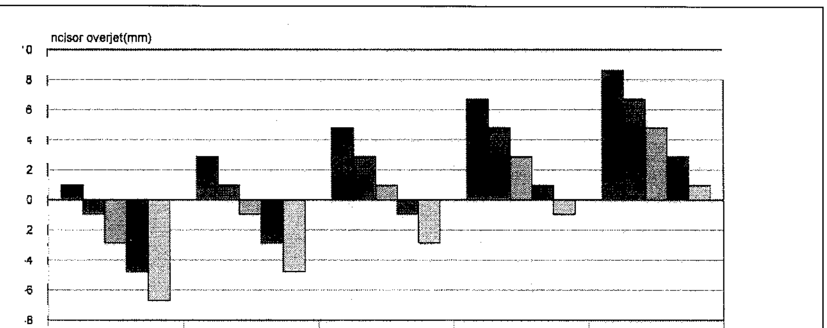
	36	37	38	39	40	41	42	43
34	0.95	1.43	1.91	2.39	2.86	3.34	3.82	4.3
35	0.48	0.95	1.43	1.91	2.39	2.86	3.34	3.82
36	0	0.48	0.95	1.43	1.91	2.39	2.86	3.34
37	-0.48	0	0.48	0.95	1.43	1.91	2.39	2.86
38	-0.95	-0.48	0	0.48	0.95	1.43	1.91	2.39
39	-1.43	-0.95	-0.48	0	0.48	0.95	1.43	1.91
40	-1.91	-1.43	-0.95	-0.48	0	0.48	0.95	1.43
41	-2.39	-1.91	-1.43	-0.95	-0.48	0	0.48	0.95

Figure 2. Incisor overjet with the sum of maxillary and mandibular anterior teeth widths varied.



	36	37	38	39	40	41	42	43
34	1.1	1.65	2.21	2.76	3.31	3.86	4.41	4.96
35	0.55	1.1	1.65	2.21	2.76	3.31	3.86	4.4
36	0	0.55	1.1	1.65	2.21	2.76	3.31	3.86
37	-0.55	0	0.55	1.1	1.65	2.21	2.76	3.3
38	-1.1	-0.55	0	0.55	1.1	1.65	2.21	2.76
39	-1.65	-1.1	-0.55	0	0.55	1.1	1.65	2.2
40	-2.21	-1.65	-1.1	-0.55	0	0.55	1.1	1.65
41	-2.76	-2.21	-1.65	-1.1	-0.55	0	0.55	1.1

Figure 3. Canine overjet with the sum of maxillary and mandibular anterior teeth widths varied.



	max -8	max -4	max 0	max 4	max 8
mand -8	0.95	2.86	4.77	6.68	8.59
mand -4	-0.95	0.95	2.86	4.77	6.68
mand 0	-2.86	-0.95	0.95	2.86	4.77
mand 4	-4.77	-2.86	-0.95	0.95	2.86
mand 8	-6.68	-4.77	-2.86	-0.95	0.95

Figure 4. Incisor overjet with variation in maxillary and mandibular crowding and spacing.

Principle 2: Increasing the perimeter of an arch increases the width and depth of that arch; decreasing the perimeter reduces its width and depth.

Principle 3: Increasing the maxillary perimeter moves the maxillary incisors forward and maxillary canines laterally, making the difference in arch depths and widths move positively (+ve) towards greater incisor and canine overjets. A perimeter decrease moves both incisor and canine overjet figures towards the negative (-ve).

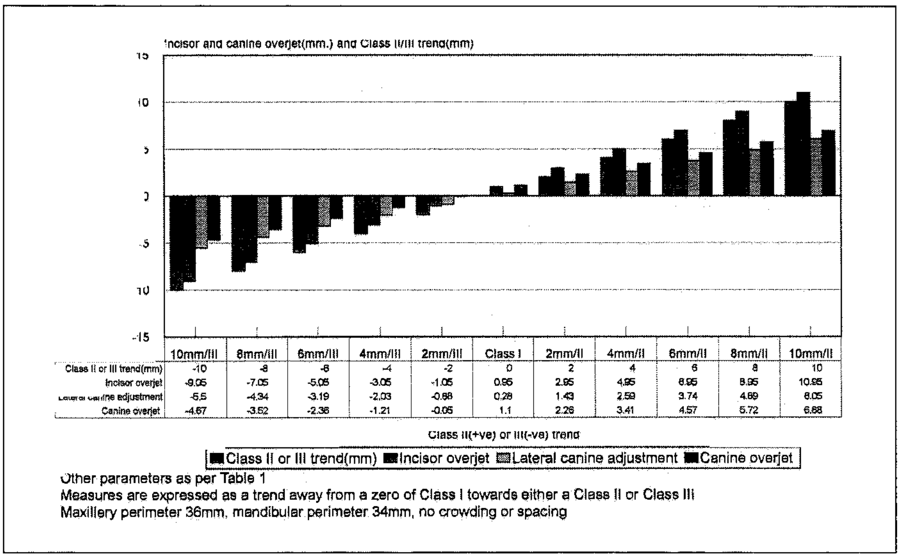
Principle 4: Increasing the mandibular perimeter width moves the mandibular incisors forward and the mandibular canines laterally. An increase in the mandibular perimeter has the opposite response to increasing the maxillary perimeter: incisor and canine overjets move towards negative. Decreasing the mandibular perimeter moves both overjets towards the positive or further positive.

Principle 5: Increasing both arch perimeters equally results in a forward movement and widening of both arches by the same amount. Incisor and canine overjets remain the same.

Arc angles

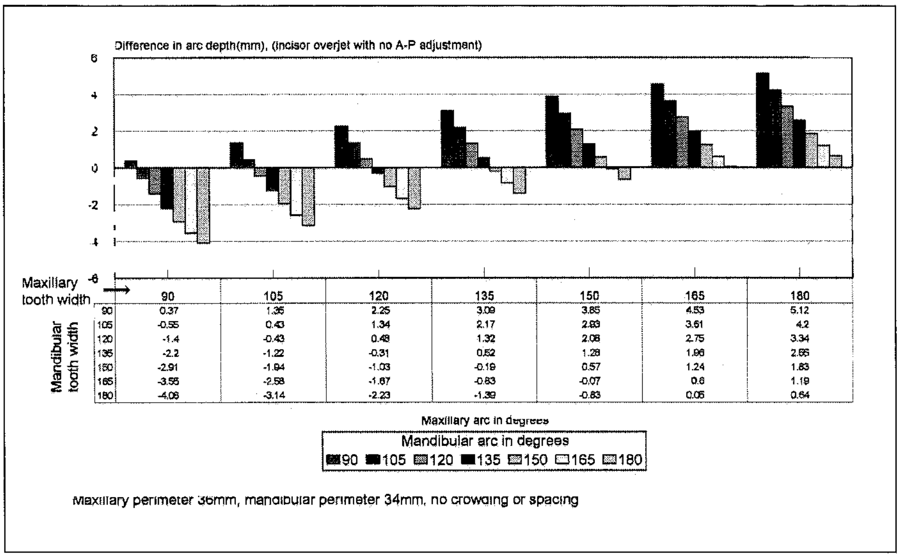
When the angles of the arcs are varied, the incisor overjet varies significantly in amount, and when canine adjustment is included, often in direction as well. When two arches differ greatly in form, widely different results can occur, with and without canine adjustment. For example: if the maxillary arch has a broader form (smaller arc angle) and is without canine adjustment, the maxillary incisors move backwards and the canines wider, reducing incisor overjet and increasing canine overjet. The reverse occurs as the maxillary arch narrows, increasing incisor overjet and decreasing canine overjet.

Arch form changes when the angle of the arc of either arch is increased, thereby progressively increasing the depth of the arch and decreasing its width. Epker and Fish[6] suggested that increasing the angles of both arcs equally (with length remaining constant) reduces the distance between arches. This premise is supported by Figures 7 and 8 but not by Figure 6. Figure 6 is the inter-arch antero-posterior difference in arch depth which slowly increases as the angles of both arcs increase at the same rate. Figure 7 shows a much greater



	10mm/III	8mm/III	6mm/III	4mm/III	2mm/III	Class I	2mm/II	4mm/II	6mm/II	8mm/II	10mm/II
Class II or III trend(mm)	-10	-8	-6	-4	-2	0	2	4	6	8	10
Incisor overjet	-9.05	-7.05	-5.05	-3.05	-1.05	0.95	2.95	4.95	6.95	8.95	10.95
Lateral canine adjustment	-5.5	-4.34	-3.19	-2.03	-0.88	0.28	1.43	2.59	3.74	4.89	6.05
Canine overjet	-4.67	-3.52	-2.36	-1.21	-0.05	1.1	2.26	3.41	4.57	5.72	6.88

Figure 5. Incisor and canine overjet incorporate a Class II and Class III trend.



	90	105	120	135	150	165	180
90	0.37	1.36	2.25	3.09	3.85	4.53	5.12
105	-0.55	0.43	1.34	2.17	2.93	3.61	4.2
120	-1.4	-0.43	0.48	1.32	2.08	2.75	3.34
135	-2.2	-1.22	-0.31	0.52	1.28	1.96	2.66
150	-2.91	-1.94	-1.03	-0.19	0.57	1.24	1.83
165	-3.55	-2.58	-1.67	-0.83	-0.07	0.6	1.19
180	-4.06	-3.14	-2.23	-1.39	-0.63	0.05	0.64

Figure 6. Incisor overjet with maxillary and mandibular arcs varied without canine adjustment.

rate of decrease of overjet if the canine is projected forward with the canine adjustment. Individual circumstances may determine which table is the more appropriate (refer also to "Canine Adjustment").

Change in the angle of the arc, with and without canine adjustment, gave the very different results shown in Figures 6 to 7. With canine adjustment and the maxillary arc fixed at 90 degrees and the angles of the mandibular arc range from 90 to 180 degrees, the incisor overjet appears to have a small range. When the angle of the mandibular arc decreases, the incisors move lingually, but canine adjustment brings the entire arch forward with a residual overjet range of about one millimetre. As the maxillary arc angle increases, it also increases the range of overjet results for various mandibular arc angles. Both Figures 6 and 7 may have application in different situations.

Principle 6: As the angle of an arch increases, the depth of the arc increases and width decreases.

Principle 7: An increase in the angle of the maxillary arc carries the maxillary incisors forward and maxillary canines palatally, thereby increasing incisor overjet (moving towards positive) and decreasing canine overjet (more negative).

Principle 8: An increase of the angle of the mandibular arc moves the mandibular incisors forward and the canines lingually; incisor overjet decreases and canine overjet increases.

Principle 9: As maxillary and mandibular arcs increase equally, both incisor (with canine adjustment) and canine overjets decrease.

Antero-posterior change

Antero-posterior adjustment of the reference plane (Figure 8) can be made. Antero-posterior adjustments can reflect an initial occlusion or a treatment scenario. Incisal overjet is an addition of the difference in antero-posterior position of the arch, plus the canine adjustment (if applicable) and the Angle Class II or Class III trend. Superimposing the antero-posterior buccal arrangement on other calculations is relatively simple; as is visualising how change in the buccal relations alters an incisal relation.

The lateral measurement is more complex. In its simpler form (as portrayed in the model), the angle of the wider of the two arches determines the rate of increase in canine overjet as the arch is projected anteriorly or posteriorly to the reference plane: for example, with the arc angles of 180 degrees, the canine overjet remains constant whatever the reference plane relationships, but as the arc angles decrease, the rate of increase in the canine overjet increases. Arch form may vary distally to the canine. A case using more sophisticated arch forms could be considered for a clinical application, however, the current simple model allows testing principles of inter-arch relations.

Principle 10: Incisor overjet increases by the same amount as an anterior adjustment for Class II and a posterior adjustment for a Class III buccal relation.

Principle 11: Canine overjet increases positively as buccal relations move towards Class II for arc angles less than 180 degrees, and canine overjet decreases with a buccal shift towards Class III.

Principle 12: If the wider arch is moved anteriorly, or the narrow arch moved posteriorly, the rate of canine overjet increase is greater, with a reduction in the angle of the wider arch.

Canine adjustment (altering the position of the maxillary arch at the level of the reference plane)

Different values were obtained when calculations were taken perpendicular to the tangent of the dental arch (at the canine in this instance), or perpendicular to the arch midline (the reference plane).

Some of the differences are presented in Figure 1. If the canine relation is considered as perpendicular to the (tangent of the) arc of an arch, the arches must be adjusted in their relation to each other. For this work, adjustments are calculated from the widest arch.

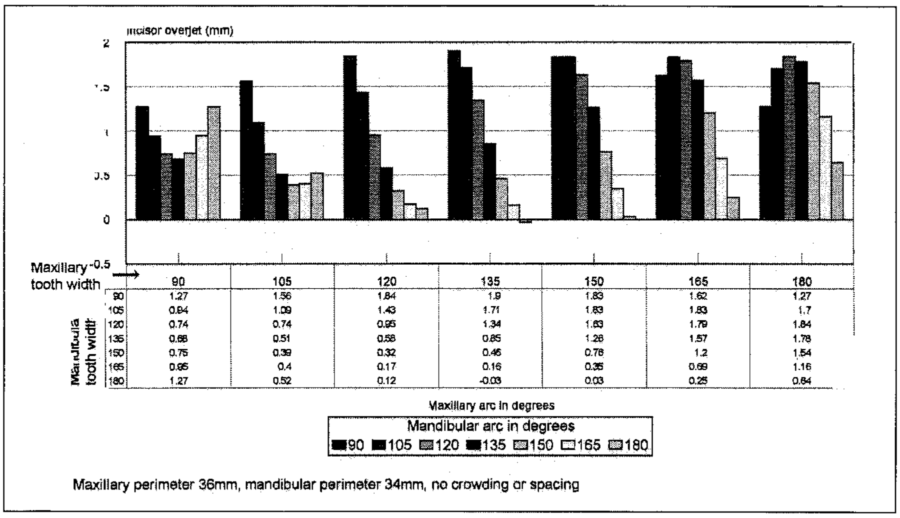


Figure 7. Incisor overjet with maxillary and mandibular arcs varied (with canine adjustment).

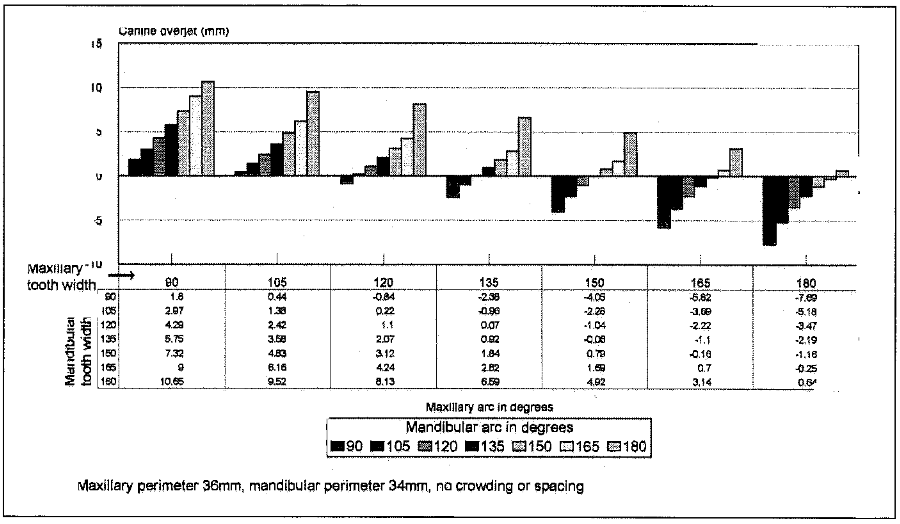


Figure 8. Canine overjet with maxillary and mandibular arc angles varied (with canine adjustment).

↑ = INCREASE – = NO CHANGE ↓ = DECREASE

	A-P ARCH DIFFERENCE	A-P CANINE ADJUSTMENT	INCISOR OVERJET	HALF THE DIFFERENCE IN LATERAL ARCH WIDTH	LATERAL CANINE ADJUSTMENT	CANINE OVERJET
MAXILLARY TOOTH WIDTH↑	↑	↑	↑	↑	↑	↑
MANDIBULAR TOOTH WIDTH↑	↓	↓	↓	↓	↓	↓
MAXILLARY SPACING↑	↑	↑	↑	↑	↑	↑
MAXILLARY CROWDING↑	↓	↓	↓	↓	↓	↓
MANDIBULAR SPACING↑	↓	↓	↓	↓	↓	↓
MANDIBULAR CROWDING↑	↑	↑	↑	↑	↑	↑
MAXILLARY AND MANDIBULAR TOOTH WIDTH OR SPACING↑ (EQUAL INTERACH CHANGE)	-	-	-	-	-	-
CLASS II TREND↑	↑	↑	↑	↑	↑	↑
CLASS III TREND↑	↓	↓	↓	↓	↓	↓
MAXILLARY ARC ANGLE↑	↓	↓	* VARIES	↓	↓	↓
MANDIBULAR ARC ANGLE↑	↓	↑	* VARIES	↑	↑	↑
MAXILLARY AND MANDIBULAR ARC ANGLES EQUALLY↑	↓	↓	↓	↓	↓	↓

* CANINE ADJUSTMENT IS NOT APPROPRIATE IN CASES WHERE THE BUCCAL RELATIONSHIP DOES NOT CHANGE WITH TRANSVERSE CHANGE. FOR THESE CASES, FIGURES WITHOUT CANINE ADJUSTMENT MAY BETTER ESTIMATE INCISOR OVERJET CHANGE.

Table IV. Change in antero-posterior and lateral interarch measures as other intra and interarch measures are altered.

Antero-posterior canine adjustment

As maxillary canine overjet increases and/or the maxillary angle of the arc decreases, the maxillary arch is projected forward relative to the reference plane.

Principle 13: Where the arch with the greater width has an angle of less than 180 degrees, canine adjustment moves this wider arch relatively forward. Where the wider arch is the maxillary arch, incisor overjet increases (moves towards or becomes more positive); if the mandibular arch is wider, incisor overjet decreases.

The greatest adjustments occur when the maxillary arc is smallest (90 degrees) and the mandibular arc is largest (180 degrees). As the angle of the maxillary arc increases, the anterior movement of the maxillary arch increases at a lesser rate. Canine adjustment displayed most variation for incisor overjet when the angles of the arc were varied.

Transverse canine adjustment

The Bonwill model has the lines of the buccal segments projected outwards at an angle of 60 degrees to the arch midline. As the maxillary arch is translated either anteriorly or posteriorly, the lateral distance between the arches changes at the level of the mandibular canine. When both maxillary and mandibular arcs have the same angular value, as both arcs increase in angular value, the canine adjustment decreases.

Principle 14: Lateral canine adjustment increases the distance of the wider arch from the narrower arch. A positive canine overjet increases, and a negative overjet becomes more negative.

Is canine adjustment valid?

Canine adjustment was calculated because it seemed more plausible that inter-digitation would be clinically measured perpendicular to the arch form; rather than perpendicular to the arch midline. There would be instances when canine adjustment is less applicable: such as, large differences in arc angles creating a crossbite with the buccal relations moving towards Class III. In such cases, use of calculations without an adjustment factor may be preferred on the understanding that inter-digitation could be influenced.

Summary of inter-arch response to altered dental measurements

A summary table (Table IV) of response to altered dental measurements is provided. The inter-arch position of the incisors in the horizontal plane can be adjusted by factors discussed previously. Several of the dental measurements may be adjusted during treatment to alter the occlusion and elicit change in the incisal and canine overjet. Table IV summarises the expected direction of change in incisor and canine inter-arch relations with changes in dental measurements.

Table IV may be used as a tool to predict changes in arch form, size and position. If large movement of the incisors in one direction is required, the combining of several of the possible movements may be necessary, and the changing of some measurements (which go against the desired movement) may be less desirable. If a small change in incisor position is required, it could be possible to alter various measurements – some increasing overjet, others reducing overjet – to allow the achievement of other treatment objectives, and not primarily overjet correction.

Controlling canine overjet has implications for crossbites and scissor-bites. There may be reasons why crossbite correction should or should not be done. Achieving arch co-ordination when arches are initially poorly co-ordinated will usually require a change in the lateral dental relations. Table IV indicates some factors which could be used to help correction, or to lessen a discrepancy.

Other arch forms

Many mathematical models have been proposed as representative of one arch. This model differs in that it compares the anterior of two arches in an approximation of occlusion. It is possible to build a model for each arch form using the perpendicular to the tangent of any arch form in the canine region and relating this to the opposing arch. Their validity is not examined here but some projections of this model and their application in other models follow. The advantage of the Bonwill model is that the position of the canine along the curve is defined and thus a reference line for a maxillary arch and a separate line for a mandibular arch can be described. With two lines described, two arches can be related to a plane.

The maxillary and mandibular arch could have significantly different forms. Change in antero-posterior buccal relations will effectively move one arch towards another whatever the geometric form. An increase in arch perimeter due to tooth width and spacing will move any arch form away from a centre, focus or other mathematical centre. If arch perimeter length is maintained, a decrease in the canine width will increase the distance from the inter-canine midpoint to the most anterior point of the arch.

Conclusion

A simple model of anterior dental relations has been developed to examine the results of varying multiple measurements simultaneously. Results from varying two measurements were calculated and presented in a matrix form; to allow a visual interpretation, graphs were connected to the numerical matrix. A series of matrices of two measurement variations were presented allowing interpretation of the influence of multiple factors.

On examination of the results, some principles have been proposed and tested within the model. The model has facilitated development of concepts involving inter-arch relations which may be employed to understand a malocclusion at the diagnostic stage or to solve a clinical problem. The stated principles and summary table are the essence of the matrices presented.

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